8-1 Multiplying and Dividing Rational Expressions

Simplify each expression.

1. $\frac{x^2-5x-24}{x^2-64}$

SOLUTION:

$$\begin{aligned}\frac{x^2-5x-24}{x^2-64} &= \frac{\cancel{(x-8)}(x+3)}{(x+8)\cancel{(x-8)}}\\ &= \frac{(x+3)}{(x+8)}\end{aligned}$$

2. $\frac{c+d}{3c^2-3d^2}$

SOLUTION:

$$\begin{aligned}\frac{c+d}{3c^2-3d^2} &= \frac{c+d}{3(c^2-d^2)}\\ &= \frac{\cancel{c+d}}{3(c-d)\cancel{(c+d)}}\\ &= \frac{1}{3(c-d)}\end{aligned}$$

3. **MULTIPLE CHOICE** Identify all values of *x* for which $\frac{x+7}{x^2-3x-28}$ is undefined.

A –7, 4

B 7, 4

C 4, –7, 7

D –4, 7

SOLUTION:

$$\frac{x+7}{x^2-3x-28}=\frac{x+7}{(x-7)(x+4)}$$

The function is undefined when the denominator tends to 0.

$$\begin{aligned}(x-7)(x+4)&=0\\ x&=7,\ -4\end{aligned}$$

The correct choice is D.

Simplify each expression.

4. $\frac{y^2+3y-40}{25-y^2}$

SOLUTION:

$$\begin{aligned}\frac{y^2+3y-40}{25-y^2}&=\frac{(y+8)(y-5)}{(5-y)(5+y)}\\ &=-\frac{(y+8)\cancel{(y-5)}}{\cancel{(y-5)}(5+y)}\\ &=-\frac{(y+8)}{(y+5)}\end{aligned}$$

5. $\dfrac{a^2x - b^2x}{by - ay}$

SOLUTION:

$$\begin{aligned}\frac{a^2x-b^2x}{by-ay} &= \frac{x(a^2-b^2)}{y(b-a)}\\ &= \frac{x(a-b)(a+b)}{y(b-a)}\\ &= -\frac{x\cancel{(b-a)}(a+b)}{y\cancel{(b-a)}}\\ &= -\frac{x(a+b)}{y}\end{aligned}$$

6. $\dfrac{27x^2y^4}{16yz^3} \cdot \dfrac{8z}{9xy^3}$

SOLUTION:

$$\frac{27x^2y^4}{16yz^3} \cdot \frac{8z}{9xy^3}$$

$$= \frac{\overset{3}{\cancel{27}} \cdot \cancel{x} \cdot x \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y}}{\underset{2}{\cancel{16}} \cdot \cancel{y} \cdot z \cdot z \cdot \cancel{z}} \cdot \frac{\overset{1}{\cancel{8}} \cdot \cancel{z}}{\underset{1}{\cancel{9}} \cdot \cancel{x} \cdot \cancel{y} \cdot \cancel{y} \cdot \cancel{y}}$$

$$= \frac{3x}{2z^2}$$

7. $\dfrac{12x^3y}{13ab^2} \div \dfrac{36xy^3}{26b}$

SOLUTION:

Invert the second expression and multiply.

$$\begin{aligned}\frac{12x^3y}{13ab^2} \div \frac{36xy^3}{26b} &= \frac{12x^3y}{13ab^2} \cdot \frac{26b}{36xy^3}\\ &= \frac{2x^2}{3aby^2}\end{aligned}$$

8. $\frac{x^2-4x-21}{x^2-6x+8}\cdot\frac{x-4}{x^2-2x-35}$

SOLUTION:

$$\begin{aligned}&\frac{x^2-4x-21}{x^2-6x+8}\cdot\frac{x-4}{x^2-2x-35}\\&=\frac{\cancel{(x-7)}(x+3)}{(x-2)\cancel{(x-4)}}\cdot\frac{\cancel{x-4}}{\cancel{(x-7)}(x+5)}\\&=\frac{(x+3)}{(x-2)(x+5)}\end{aligned}$$

9. $\frac{a^2-b^2}{3a^2-6a+3}\div\frac{4a+4b}{a^2-1}$

SOLUTION:

Invert the second expressions and multiply.

$$\begin{aligned}&\frac{a^2-b^2}{3a^2-6a+3}\div\frac{4a+4b}{a^2-1}\\&=\frac{a^2-b^2}{3a^2-6a+3}\cdot\frac{a^2-1}{4a+4b}\\&=\frac{(a+b)(a-b)}{3(a^2-2a+1)}\cdot\frac{(a-1)(a+1)}{4(a+b)}\\&=\frac{(a+b)(a-b)}{3(a-1)^2}\cdot\frac{(a-1)(a+1)}{4(a+b)}\\&=\frac{(a-b)(a+1)}{12(a-1)}\end{aligned}$$

10. $\dfrac{\frac{a^3b^3}{xy^4}}{\frac{a^2b}{x^2y}}$

SOLUTION:

Invert the second expressions and multiply.

$$\begin{aligned}\frac{\frac{a^3b^3}{xy^4}}{\frac{a^2b}{x^2y}} &= \frac{a^3b^3}{xy^4}\cdot\frac{x^2y}{a^2b}\\ &= \frac{ab^2x}{y^3}\end{aligned}$$

11. $\dfrac{\frac{4x}{x+6}}{\frac{x^2-3x}{x^2+3x-18}}$

SOLUTION:

Invert the second expressions and multiply.

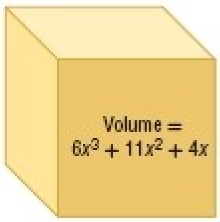
$$\begin{aligned}\frac{\frac{4x}{x+6}}{\frac{x^2-3x}{x^2+3x-18}} &= \frac{4x}{x+6}\cdot\frac{x^2+3x-18}{x^2-3x}\\ &= \frac{4x}{x+6}\cdot\frac{(x+6)(x-3)}{x(x-3)}\\ &= 4\end{aligned}$$

12. **CCSS SENSE-MAKING** The volume of a shipping container in the shape of a rectangular prism can be represented by the polynomial $6x^3 + 11x^2 + 4x$, where the height is x.

a. Find the length and width of the container.

b. Find the ratio of the three dimensions of the container when $x = 2$.

c. Will the ratio of the three dimensions be the same for all values of x?



SOLUTION:

a. Divide the volume by the height of the container to find the length and width of the container

$$\begin{aligned}\frac{6x^3+11x^2+4x}{x} &= \frac{x(6x^2+11x+4)}{x}\\ &= 6x^2+11x+4\\ &= 6x^2+8x+3x+4\\ &= 2x(3x+4)+1(3x+4)\\ &= (2x+1)(3x+4)\end{aligned}$$

The length is $(2x + 1)$ and width is $(3x + 4)$.

b. Substitute $x = 2$ in the expressions for height, length and width.

Height: $x = 2$
Length: $2(2)+1=5$
Width: $3(2)+4=10$

c. No because the expressions for height, length and width are different.

Simplify each expression.

13. $\dfrac{x(x-3)(x+6)}{x^2+x-12}$

SOLUTION:

$$\begin{aligned}\frac{x(x-3)(x+6)}{x^2+x-12} &= \frac{x(x-3)(x+6)}{(x+4)(x-3)}\\ &= \frac{x(x+6)}{(x+4)}\end{aligned}$$

14. $\dfrac{y^2(y^2+3y+2)}{2y(y-4)(y+2)}$

SOLUTION:

$$\begin{aligned}\frac{y^2(y^2+3y+2)}{2y(y-4)(y+2)} &= \frac{y^2(y+1)(y+2)}{2y(y-4)(y+2)}\\ &= \frac{y(y+1)}{2(y-4)}\end{aligned}$$

15. $\dfrac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)}$

SOLUTION:

$$\begin{aligned}\frac{(x^2-9)(x^2-z^2)}{4(x+z)(x-3)} &= \frac{(x+3)(x-3)(x-z)(x+z)}{4(x+z)(x-3)}\\ &= \frac{(x+3)(x-z)}{4}\end{aligned}$$

16. $\dfrac{(x^2-16x+64)(x+2)}{(x^2-64)(x^2-6x-16)}$

SOLUTION:

$$\begin{aligned}\frac{(x^2-16x+64)(x+2)}{(x^2-64)(x^2-6x-16)} &= \frac{(x-8)^2(x+2)}{(x+8)(x-8)(x-8)(x+2)}\\ &= \frac{1}{x+8}\end{aligned}$$

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17. $\frac{x^2(x+2)(x-4)}{6x(x^2+x-20)}$

SOLUTION:

$$\frac{x^2(x+2)(x-4)}{6x(x^2+x-20)} = \frac{x^2(x+2)(x-4)}{6x(x+5)(x-4)}$$
$$= \frac{x(x+2)}{6(x+5)}$$

18. $\frac{3y(y-8)(y^2+2y-24)}{15y^2(y^2-12y+32)}$

SOLUTION:

$$\frac{3y(y-8)(y^2+2y-24)}{15y^2(y^2-12y+32)}$$
$$= \frac{3y(y-8)(y+6)(y-4)}{15y^2(y-4)(y-8)}$$
$$= \frac{(y+6)}{5y}$$

19. **MULTIPLE CHOICE** Identify all values of x for which $\frac{(x-3)(x+6)}{(x^2-7x+12)(x^2-36)}$ is undefined.

F 3, –6

G 4, 6

H –6, 6

J –6, 3, 4, 6

SOLUTION:

$$\frac{(x-3)(x+6)}{(x^2-7x+12)(x^2-36)}$$
$$= \frac{(x-3)(x+6)}{(x-4)(x-3)(x-6)(x+6)}$$

Therefore, the function is undefined for $x = -6, 3, 4, 6$. So, the correct choice is J.

Simplify each expression.

20. $\frac{x^2-5x-14}{28+3x-x^2}$

SOLUTION:

$$\begin{aligned}\frac{x^2-5x-14}{28+3x-x^2} &= \frac{(x-7)(x+2)}{-(x^2-3x-28)} \\ &= \frac{(x-7)(x+2)}{-(x+4)(x-7)} \\ &= -\frac{(x+2)}{(x+4)}\end{aligned}$$

21. $\frac{x^3-9x^2}{x^2-3x-54}$

SOLUTION:

$$\begin{aligned}\frac{x^3-9x^2}{x^2-3x-54} &= \frac{x^2(x-9)}{(x-9)(x+6)} \\ &= \frac{x^2}{x+6}\end{aligned}$$

22. $\frac{(x-4)(x^2+2x-48)}{(36-x^2)(x^2+4x-32)}$

SOLUTION:

$$\frac{(x-4)(x^2+2x-48)}{(36-x^2)(x^2+4x-32)}$$

$$= \frac{(x-4)(x+8)(x-6)}{-(x-6)(x+6)(x+8)(x-4)}$$

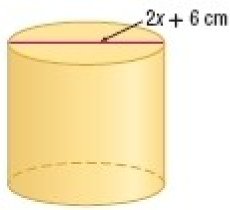
$$= -\frac{1}{(x+6)}$$

23. $\frac{16-c^2}{c^2+c-20}$

SOLUTION:

$$\begin{aligned}\frac{16-c^2}{c^2+c-20} &= \frac{-(c-4)(c+4)}{(c+5)(c-4)} \\ &= -\frac{c+4}{c+5}\end{aligned}$$

24. **GEOMETRY** The cylinder has a volume of $(x + 3)(x^2 - 3x - 18)\pi$ cubic centimeters. Find the height of the cylinder.



SOLUTION:

$$\begin{aligned}\pi r^2 h &= (x+3)(x^2-3x-18)\pi \\ \pi(x+3)^2 h &= (x+3)(x^2-3x-18)\pi \\ h &= \frac{x^2-3x-18}{x+3} \\ h &= \frac{(x-6)(x+3)}{(x+3)} \\ h &= x-6\end{aligned}$$

Therefore, the height of the cylinder is $(x - 6)$ centimeters.

Simplify each expression.

25. $\frac{3ac^3f^3}{8a^2bcf^4} \cdot \frac{12ab^2c}{18ab^3c^2f}$

SOLUTION:

$$\frac{3ac^3f^3}{8a^2bcf^4} \cdot \frac{12ab^2c}{18ab^3c^2f} = \frac{c}{4ab^2f^2}$$

26. $\frac{14xy^2z^3}{21w^4x^2yz} \cdot \frac{7wxyz}{12w^2y^3z}$

SOLUTION:

$$\frac{14xy^2z^3}{21w^4x^2yz} \cdot \frac{7wxyz}{12w^2y^3z} = \frac{7z^2}{18w^5y}$$

27. $\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2}$

SOLUTION:

Invert the second fraction and multiply.

$$\begin{aligned}\frac{64a^2b^5}{35b^2c^3f^4} \div \frac{12a^4b^3c}{70abcf^2} &= \frac{64a^2b^5}{35b^2c^3f^4} \times \frac{70abcf^2}{12a^4b^3c} \\ &= \frac{32b}{3ac^3f^2}\end{aligned}$$

28. $\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2}$

SOLUTION:

Invert the second fraction and multiply.

$$\begin{aligned}\frac{9x^2yz}{5z^4} \div \frac{12x^4y^2}{50xy^4z^2} &= \frac{9x^2yz}{5z^4} \cdot \frac{50xy^4z^2}{12x^4y^2} \\ &= \frac{15y^3}{2xz}\end{aligned}$$

29. $\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3}$

SOLUTION:

$$\frac{15a^2b^2}{21ac} \cdot \frac{14a^4c^2}{6ab^3} = \frac{5a^4c}{3b}$$

8-1 Multiplying and Dividing Rational Expressions

30. $\frac{14c^2f^5}{9a^2} \div \frac{35cf^4}{18ab^3}$

SOLUTION:

Invert the second fraction and multiply.

$$\frac{14c^2f^5}{9a^2} \div \frac{35cf^4}{18ab^3} = \frac{14c^2f^5}{9a^2} \times \frac{18ab^3}{35cf^4}$$
$$= \frac{4cfb^3}{5a}$$

31. $\frac{y^2+8y+15}{y-6} \cdot \frac{y^2-9y+18}{y^2-9}$

SOLUTION:

$$\frac{y^2+8y+15}{y-6} \cdot \frac{y^2-9y+18}{y^2-9}$$
$$= \frac{(y+5)(y+3)}{y-6} \cdot \frac{(y-6)(y-3)}{(y-3)(y+3)}$$
$$= y+5$$

32. $\frac{c^2-6c-16}{c^2-d^2} \div \frac{c^2-8c}{c+d}$

SOLUTION:

Invert the second fraction and multiply.

$$\frac{c^2-6c-16}{c^2-d^2} \div \frac{c^2-8c}{c+d}$$
$$= \frac{c^2-6c-16}{c^2-d^2} \cdot \frac{c+d}{c^2-8c}$$
$$= \frac{(c-8)(c+2)}{(c-d)(c+d)} \cdot \frac{c+d}{c(c-8)}$$
$$= \frac{(c+2)}{c(c-d)}$$

33. $\frac{x^2+9x+20}{8x+16} \cdot \frac{4x^2+16x+16}{x^2-25}$

SOLUTION:

$$\begin{aligned}
&\frac{x^2+9x+20}{8x+16} \cdot \frac{4x^2+16x+16}{x^2-25} \\
&= \frac{(x+4)(x+5)}{8(x+2)} \cdot \frac{4(x^2+4x+4)}{(x-5)(x+5)} \\
&= \frac{(x+4)}{2} \cdot \frac{(x+2)}{(x-5)} \\
&= \frac{(x+4)(x+2)}{2(x-5)}
\end{aligned}$$

34. $\frac{3a^2+6a+3}{a^2-3a-10} \div \frac{12a^2-12}{a^2-4}$

SOLUTION:

Invert the second fraction and multiply.

$$\begin{aligned}
&\frac{3a^2+6a+3}{a^2-3a-10} \div \frac{12a^2-12}{a^2-4} \\
&= \frac{3a^2+6a+3}{a^2-3a-10} \cdot \frac{a^2-4}{12a^2-12} \\
&= \frac{3(a^2+2a+1)}{a^2-3a-10} \cdot \frac{a^2-4}{12(a^2-1)} \\
&= \frac{3(a+1)^2}{(a-5)(a+2)} \cdot \frac{(a-2)(a+2)}{12(a-1)(a+1)} \\
&= \frac{(a+1)}{(a-5)} \cdot \frac{(a-2)}{4(a-1)} \\
&= \frac{(a+1)(a-2)}{4(a-5)(a-1)}
\end{aligned}$$

35. $\dfrac{\frac{x^2-9}{6x-12}}{\frac{x^2+10x+21}{x^2-x-2}}$

SOLUTION:

Invert the fraction in the denominator and multiply.

$$\begin{aligned}\frac{\frac{x^2-9}{6x-12}}{\frac{x^2+10x+21}{x^2-x-2}} &= \frac{x^2-9}{6x-12}\cdot\frac{x^2-x-2}{x^2+10x+21}\\ &= \frac{(x+3)(x-3)}{6(x-2)}\cdot\frac{(x-2)(x+1)}{(x+3)(x+7)}\\ &= \frac{(x-3)(x+1)}{6(x+7)}\end{aligned}$$

36. $\dfrac{\frac{y-x}{z^3}}{\frac{x-y}{6z^2}}$

SOLUTION:

Invert the fraction in the denominator and multiply.

$$\begin{aligned}\frac{\frac{y-x}{z^3}}{\frac{x-y}{6z^2}} &= \frac{y-x}{z^3}\cdot\frac{6\cancel{z}^2}{x-y}\\ &= \frac{-(x-y)}{z^3}\cdot\frac{6\cancel{z}^2}{x-y}\\ &= -\frac{6}{z}\end{aligned}$$

37. $\dfrac{\frac{a^2-b^2}{b^3}}{\frac{b^2-ab}{a^2}}$

SOLUTION:

Invert the fraction in the denominator and multiply.

$$\begin{aligned}\frac{\frac{a^2-b^2}{b^3}}{\frac{b^2-ab}{a^2}} &= \frac{a^2-b^2}{b^3}\cdot\frac{a^2}{b^2-ab}\\ &= \frac{(a-b)(a+b)}{b^3}\cdot\frac{a^2}{-b(a-b)}\\ &= -\frac{a^2(a+b)}{b^4}\end{aligned}$$

38. $\dfrac{\frac{x-y}{a+b}}{\frac{x^2-y^2}{b^2-a^2}}$

SOLUTION:

Invert the fraction in the denominator and multiply.

$$\begin{aligned}\frac{\frac{x-y}{a+b}}{\frac{x^2-y^2}{b^2-a^2}} &= \frac{x-y}{a+b}\cdot\frac{b^2-a^2}{x^2-y^2}\\ &= \frac{x-y}{a+b}\cdot\frac{(b-a)(b+a)}{(x-y)(x+y)}\\ &= \frac{b-a}{x+y}\end{aligned}$$

39. **CCSS REASONING** At the end of her high school soccer career, Ashley had made 33 goals out of 121 attempts.

a. Write a ratio to represent the ratio of the number of goals made to goals attempted by Ashley at the end of her high school career.

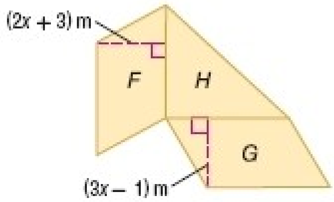
b. Suppose Ashley attempted *a* goals and made *m* goals during her first year at college. Write a rational expression to represent the ratio of the number of career goals made to the number of career goals attempted at the end of her first year in college.

SOLUTION:

a. $\frac{33}{121}$

b. $\frac{33+m}{121+a}$

40. **GEOMETRY** Parallelogram *F* has an area of $8x^2 + 10x - 3$ square meters and a height of $2x + 3$ meters. Parallelogram *G* has an area of $6x^2 + 13x - 5$ square meters and a height of $3x - 1$ meters. Find the area of right triangle *H*.



SOLUTION:

Find the base of the parallelogram *F*.

$$\frac{8x^2+10x-3}{(2x+3)} = \frac{(2x+3)(4x-1)}{(2x+3)} = (4x-1)$$

The base of the parallelogram *F* is $(4x - 1)$ meters.

Find the base of the parallelogram *G*.

$$\frac{6x^2+13x-5}{3x-1} = \frac{(2x+5)(3x-1)}{3x-1} = (2x+5)$$

The base of the parallelogram *G* is $(2x + 5)$ meters.

Find the area of the right triangle *H*.

Area of the right triangle *H*

$$= \frac{1}{2}(2x+5)(4x-1)$$
$$= \frac{1}{2}\left(8x^2 - 2x + 20x - 5\right)$$
$$= \frac{1}{2}\left(8x^2 + 18x - 5\right)$$

The area of the right triangle *H* is $\frac{1}{2}\left(8x^2+18x-5\right)\ \text{m}^2$.

41. **POLLUTION** The thickness of an oil spill from a ruptured pipe on a rig is modeled by the function $T(x) = \frac{0.4(x^2 - 2x)}{x^3 + x^2 - 6x}$, where T is the thickness of the oil slick in meters and x is the distance from the rupture in meters.

a. Simplify the function.

b. How thick is the slick 100 meters from the rupture?

SOLUTION:

a.

$$\begin{aligned} T(x) &= \frac{0.4\left(x^2 - 2x\right)}{x^3 + x^2 - 6x} \\ &= \frac{0.4x(x-2)}{x\left(x^2 + x - 6\right)} \\ &= \frac{0.4x(x-2)}{x(x+3)(x-2)} \\ &= \frac{0.4}{(x+3)} \end{aligned}$$

b. Substitute $x = 100$ in the function $T(x) = \frac{0.4}{x+3}$.

$$\begin{aligned} T(100) &= \frac{0.4}{100+3} \\ &= \frac{0.4}{103} \\ &\approx 0.0039 \end{aligned}$$

The slick is about 0.0039 meters or 3.9 mm thick.

Simplify each expression.

42. $\frac{x^2-16}{3x^3+18x^2+24x} \cdot \frac{x^3-4x}{2x^2-7x-4}$

SOLUTION:

$$\frac{x^2-16}{3x^3+18x^2+24x} \cdot \frac{x^3-4x}{2x^2-7x-4}$$
$$= \frac{(x+4)(x-4)}{3x(x^2+6x+8)} \cdot \frac{x(x^2-4)}{2x^2-7x-4}$$
$$= \frac{(x+4)(x-4)}{3x(x+2)(x+4)} \cdot \frac{x(x+2)(x-2)}{(2x+1)(x-4)}$$
$$= \frac{(x-2)}{3(2x+1)}$$

43. $\frac{3x^2-17x-6}{4x^2-20x-24} \div \frac{6x^2-7x-3}{2x^2-x-3}$

SOLUTION:

$$\frac{3x^2-17x-6}{4x^2-20x-24} \div \frac{6x^2-7x-3}{2x^2-x-3}$$
$$= \frac{3x^2-17x-6}{4x^2-20x-24} \cdot \frac{2x^2-x-3}{6x^2-7x-3}$$
$$= \frac{3x^2-17x-6}{4(x^2-5x-6)} \cdot \frac{2x^2-x-3}{6x^2-7x-3}$$
$$= \frac{(3x+1)(x-6)}{4(x-6)(x+1)} \cdot \frac{(2x-3)(x+1)}{(3x+1)(2x-3)}$$
$$= \frac{1}{4}$$

8-1 Multiplying and Dividing Rational Expressions

44. $\frac{9-x^2}{x^2-4x-21}\cdot\left(\frac{2x^2+7x+3}{2x^2-15x+7}\right)^{-1}$

SOLUTION:

$$\frac{9-x^2}{x^2-4x-21}\cdot\left(\frac{2x^2+7x+3}{2x^2-15x+7}\right)^{-1}$$
$$=\frac{9-x^2}{x^2-4x-21}\cdot\frac{2x^2-15x+7}{2x^2+7x+3}$$
$$=\frac{(3-x)(3+x)}{(x-7)(x+3)}\cdot\frac{(2x-1)(x-7)}{(2x+1)(x+3)}$$
$$=\frac{(3-x)(2x-1)}{(x+3)(2x+1)}$$

45. $\left(\frac{2x^2+2x-12}{x^2+4x-5}\right)^{-1}\cdot\frac{2x^3-8x}{x^2-2x-35}$

SOLUTION:

$$\left(\frac{2x^2+2x-12}{x^2+4x-5}\right)^{-1}\cdot\frac{2x^3-8x}{x^2-2x-35}$$
$$=\frac{x^2+4x-5}{2x^2+2x-12}\cdot\frac{2x^3-8x}{x^2-2x-35}$$
$$=\frac{x^2+4x-5}{2(x^2+x-6)}\cdot\frac{2x(x^2-4)}{x^2-2x-35}$$
$$=\frac{(x+5)(x-1)}{2(x+3)(x-2)}\cdot\frac{2x(x+2)(x-2)}{(x-7)(x+5)}$$
$$=\frac{x(x+2)(x-1)}{(x+3)(x-7)}$$

8-1 Multiplying and Dividing Rational Expressions

46. $\left(\dfrac{3xy^3z}{2a^2bc^2}\right)^3 \cdot \dfrac{16a^4b^3c^5}{15x^7yz^3}$

SOLUTION:

$$\begin{aligned}\left(\frac{3xy^3z}{2a^2bc^2}\right)^3 \cdot \frac{16a^4b^3c^5}{15x^7yz^3} &= \frac{27x^3y^9z^3}{8a^6b^3c^6} \cdot \frac{16a^4b^3c^5}{15x^7yz^3} \\ &= \frac{9y^8}{a^2c} \cdot \frac{2}{5x^4} \\ &= \frac{18y^8}{5a^2cx^4}\end{aligned}$$

47. $\dfrac{20x^2y^6z^{-2}}{3a^3c^2} \cdot \left(\dfrac{16x^3y^3}{9acz}\right)^{-1}$

SOLUTION:

$$\begin{aligned}\frac{20x^2y^6z^{-2}}{3a^3c^2} \cdot \left(\frac{16x^3y^3}{9acz}\right)^{-1} &= \frac{20x^2y^6z^{-2}}{3a^3c^2} \cdot \frac{9acz}{16x^3y^3} \\ &= \frac{15y^3}{4a^2cxz}\end{aligned}$$

48. $\left(\dfrac{2xy^3}{3abc}\right)^{-2} \div \dfrac{6a^2b}{x^2y^4}$

SOLUTION:

$$\begin{aligned}\left(\frac{2xy^3}{3abc}\right)^{-2} \div \frac{6a^2b}{x^2y^4} &= \frac{9a^2b^2c^2}{4x^2y^6} \cdot \frac{x^2y^4}{6a^2b} \\ &= \frac{3bc^2}{8y^2}\end{aligned}$$

49. $\dfrac{\dfrac{8x^2-10x-3}{10x^2+35x-20}}{\dfrac{2x^2+x-6}{4x^2+18x+8}}$

SOLUTION:

$$\begin{aligned}
&\frac{\frac{8x^2-10x-3}{10x^2+35x-20}}{\frac{2x^2+x-6}{4x^2+18x+8}}\\
&=\frac{8x^2-10x-3}{10x^2+35x-20}\cdot\frac{4x^2+18x+8}{2x^2+x-6}\\
&=\frac{(4x+1)(2x-3)}{5(2x-1)(x+4)}\cdot\frac{2(2x+1)(x+4)}{(x+2)(2x-3)}\\
&=\frac{2(4x+1)(2x+1)}{5(2x-1)(x+2)}
\end{aligned}$$

50. $\dfrac{\dfrac{2x^2+7x-30}{-6x^2+13x+5}}{\dfrac{4x^2+12x-72}{3x^2-11x-4}}$

SOLUTION:

$$\begin{aligned}
&\frac{\frac{2x^2+7x-30}{-6x^2+13x+5}}{\frac{4x^2+12x-72}{3x^2-11x-4}}\\
&=\frac{2x^2+7x-30}{-6x^2+13x+5}\times\frac{3x^2-11x-4}{4x^2+12x-72}\\
&=\frac{(2x-5)(x+6)}{-(3x+1)(2x-5)}\times\frac{(3x+1)(x-4)}{4(x+6)(x-3)}\\
&=-\frac{(x-4)}{4(x-3)}
\end{aligned}$$

51. $\dfrac{\dfrac{4x^2-1}{3x^3-6x^2-24x}}{\dfrac{12x^2+12x-9}{-2x^2+5x+12}}$

SOLUTION:

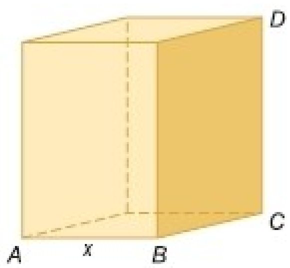
$$\begin{aligned}
&\frac{\frac{4x^2-1}{3x^3-6x^2-24x}}{\frac{12x^2+12x-9}{-2x^2+5x+12}}\\
&=\frac{4x^2-1}{3x^3-6x^2-24x}\cdot\frac{-2x^2+5x+12}{12x^2+12x-9}\\
&=\frac{(2x+1)(2x-1)}{3x(x-4)(x+2)}\cdot\frac{-(2x+3)(x-4)}{3(2x+1)(2x-3)}\\
&=-\frac{(2x-1)}{9x(x+2)}
\end{aligned}$$

52. **GEOMETRY** The area of the base of the rectangular prism at the right is 20 square centimeters.

a. Find the length of $\overline{BC}$ in terms of *x*.

b. If *DC* = 3*BC*, determine the area of the shaded region in terms of *x*.

c. Determine the volume of the prism in terms of *x*.



SOLUTION:

a. The length of $\overline{BC}$ is $\frac{20}{x}$ centimeters.

b. The length of $\overline{DC}$ in terms of *x* is $3\left(\frac{20}{x}\right)$ or $\frac{60}{x}$.

Therefore, the area of the shaded region is $\frac{20}{x} \cdot \frac{60}{x}$ or $\frac{1200}{x^2}$ square centimeters.

c. The height of the prism is $\frac{60}{x}$ centimeters.

$$\begin{aligned} \text{Volume of the prism} &= \text{Base area} \times \text{height} \\ &= 20 \cdot \frac{60}{x} \\ &= \frac{1200}{x} \end{aligned}$$

8-1 Multiplying and Dividing Rational Expressions

Simplify each expression.

53. $\frac{x^2+4x-32}{2x^2+9x-5}\cdot\frac{3x^2-75}{3x^2-11x-4}\div\frac{6x^2-18x-60}{x^3-4x}$

SOLUTION:

$$\frac{x^2+4x-32}{2x^2+9x-5}\cdot\frac{3x^2-75}{3x^2-11x-4}\div\frac{6x^2-18x-60}{x^3-4x}$$
$$=\frac{x^2+4x-32}{2x^2+9x-5}\cdot\frac{3x^2-75}{3x^2-11x-4}\cdot\frac{x^3-4x}{6x^2-18x-60}$$
$$=\frac{(x+8)(x-4)}{(2x-1)(x+5)}\cdot\frac{3(x+5)(x-5)}{(3x+1)(x-4)}\cdot\frac{x(x+2)(x-2)}{6(x-5)(x+2)}$$
$$=\frac{x(x+8)(x-2)}{2(3x+1)(2x-1)}$$

54. $\frac{8x^2+10x-3}{3x^2-12x-36}\div\frac{2x^2-5x-12}{3x^2-17x-6}\cdot\frac{4x^2+3x-1}{4x^2-40x+24}$

SOLUTION:

$$\frac{8x^2+10x-3}{3x^2-12x-36}\div\frac{2x^2-5x-12}{3x^2-17x-6}\cdot\frac{4x^2+3x-1}{4x^2-40x+24}$$
$$=\frac{8x^2+10x-3}{3(x^2-4x-12)}\times\frac{3x^2-17x-6}{2x^2-5x-12}\cdot\frac{4x^2+3x-1}{4(x^2-10x+62)}$$
$$=\frac{(4x-1)(2x+3)}{3(x-6)(x+2)}\times\frac{(x-6)(3x+1)}{(2x+3)(x-4)}\cdot\frac{(4x-1)(x+1)}{4(x^2-10x+6)}$$
$$=\frac{(4x-1)^2(3x+1)(x+1)}{12(x+2)(x-4)(x^2-10x+6)}$$

55. $\frac{4x^2-9x-9}{3x^2+6x-18}\div\frac{-2x^2+5x+3}{x^2-4x-32}\div\frac{8x^2+10x+3}{6x^2-6x-12}$

SOLUTION:

$$\frac{4x^2-9x-9}{3x^2+6x-18}\div\frac{-2x^2+5x+3}{x^2-4x-32}\div\frac{8x^2+10x+3}{6x^2-6x-12}$$
$$=\frac{4x^2-9x-9}{3x^2+6x-18}\cdot\frac{x^2-4x-32}{-2x^2+5x+3}\cdot\frac{6x^2-6x-12}{8x^2+10x+3}$$
$$=\frac{(4x+3)(x-3)}{3(x^2+2x-6)}\cdot\frac{(x-8)(x+4)}{-(x-3)(2x+1)}\cdot\frac{6(x-2)(x+1)}{(2x+1)(4x+3)}$$
$$=-\frac{2(x-8)(x+4)(x-2)(x+1)}{(2x+1)^2(x^2+2x-6)}$$

56. **CCSS PERSEVERANCE** Use the formula $d = rt$ and the following information.
An airplane is traveling at a rate r of 500 miles per hour for a time t of $(6 + x)$ hours. A second airplane travels at the rate of $(540 + 90x)$ miles per hour for a time t of 6 hours.

a. Write a rational expression to represent the ratio of the distance d traveled by the first airplane to the distance d traveled by the second airplane.

b. Simplify the rational expression. What does this expression tell you about the distances traveled by the two airplanes?

c. Under what condition is the rational expression undefined? Describe what this condition would tell you about the two airplanes.

SOLUTION:

a. $\frac{500(6+x)}{(540+90x)(6)}$

b.

$$\frac{500(6+x)}{(540+90x)6} = \frac{500(6+x)}{90(6+x)(6)}$$
$$= \frac{500}{540}$$
$$= \frac{50}{54}$$
$$= \frac{25}{27\partial}$$

The second airplane travels a bit farther than the first airplane.

c. $x = -6$. Sample answer: When $x = -6$, the first airplane would travel for 0 hours and the second airplane would travel at a rate of 0 miles per hour.

57. **TRAINS** Trying to get into a train yard one evening, all of the trains are backed up for 2 miles along a system of tracks. Assume that each car occupies an average of 75 feet of space on a track and that the train yard has 5 tracks.

a. Write an expression that could be used to determine the number of train cars involved in the backup.

b. How many train cars are involved in the backup?

c. Suppose that there are 8 attendants doing safety checks on each car, and it takes each vehicle an average of 45 seconds for each check. Approximately how many hours will it take for all the vehicles in the backup to exit?

SOLUTION:

a. $5 \text{ tracks} \cdot \frac{2 \text{ miles}}{1 \text{ track}} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} \cdot \frac{1 \text{ car}}{75 \text{ feet}}$

b. Simplify the expression.

$$5\text{tracks} \cdot \frac{2\text{miles}}{1\text{track}} \cdot \frac{5280\text{feet}}{1\text{mile}} \cdot \frac{1\text{car}}{75\text{feet}}$$
$$= \frac{5(2)(5280)}{75} \text{ cars}$$
$$= \frac{52800}{75}$$
$$= 704$$

Therefore, 704 train cars are involved in the backup.

c.

$$8(704)(45) = 253440 \text{ seconds}$$
$$= \frac{253440}{60(60)} \text{ hour}$$
$$= 70.4 \text{ hour}$$

Therefore, it will take about 70.4 hours for all the vehicles in the backup to exit.

58. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the graph of a rational function.

a. ALGEBRAIC Simplify $\frac{x^2 - 5x + 4}{x - 4}$.

b. TABULAR Let $f(x) = \frac{x^2 - 5x + 4}{x - 4}$. Use the expression you wrote in part **a** to write the related function $g(x)$. Use a graphing calculator to make a table for both functions for $0 \le x \le 10$.

c. ANALYTICAL What are $f(4)$ and $g(4)$? Explain the significance of these values.

d. GRAPHICAL Graph the functions on the graphing calculator. Use the **TRACE** function to investigate each graph, using the ▼ and ▲ keys to switch from one graph to the other. Compare and contrast the graphs.

e. VERBAL What conclusions can you draw about the expressions and the functions?

SOLUTION:

a.

$$\frac{x^2-5x+4}{x-4} = \frac{(x-4)(x-1)}{x-4}$$
$$= x-1$$

b.

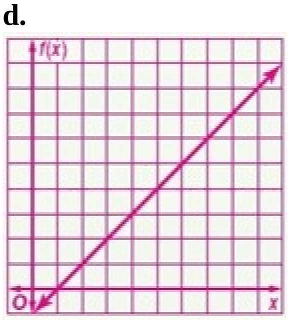
x	0	1	2	3
$f(x)$	−1	0	1	2
$g(x)$	−1	0	1	2

x	4	5	6	7
$f(x)$	ERR	4	5	6
$g(x)$	3	4	5	6

x	8	9	10
$f(x)$	7	8	9
$g(x)$	7	8	9

c. $f(4)$ results in an error because the function is undefined at $x = 4$. $g(4) = 3$

d.



The graphs appear to be the same on the graphing calculator. But $f(x)$ is undefined for $f(4)$ and $g(4) = 3$.

e. The expressions and functions are equivalent except for $x = 4$.

59. **REASONING** Compare and contrast $\frac{(x-6)(x+2)(x+3)}{x+3}$ and $(x-6)(x+2)$.

SOLUTION:
Sample answer: The two expressions are equivalent, except that the rational expression is undefined at $x = 3$.

60. **CCSS CRITIQUE** Troy and Beverly are simplifying $\frac{x+y}{x-y} \div \frac{4}{y-x}$. Is either of them correct? Explain your reasoning.

Troy

$$\frac{x+y}{x-y} \div \frac{4}{y-x} = \frac{x-y}{x+y} \cdot \frac{4}{y-x}$$
$$= \frac{-4}{x+y}$$

Beverly

$$\frac{x+y}{x-y} \div \frac{4}{y-x} = \frac{x+y}{x-y} \cdot \frac{y-x}{4}$$
$$= -\frac{x+y}{4}$$

SOLUTION:
Sample answer: Beverly; Troy's mistake was multiplying by the reciprocal of the dividend instead of the divisor.

61. **CHALLENGE** Find the value that makes the following statement true.

$$\frac{x-6}{x+3} \cdot \frac{?}{x-6} = x-2$$

SOLUTION:

$$\begin{aligned}\frac{x-6}{x+3} \cdot \frac{x^2+x-6}{x-6} &= \frac{x^2+x-6}{x+3} \\ &= \frac{(x+3)(x-2)}{(x+3)} \\ &= x-2\end{aligned}$$

Therefore, the expression $x^2 + x - 6$ makes the statement true.

62. **WHICH ONE DOESN'T BELONG?** Identify the expression that does not belong with the other three. Explain your reasoning.

$$\boxed{\frac{1}{x-1}}$$

$$\boxed{\frac{x^2+3x+2}{x-5}}$$

$$\boxed{\frac{x+1}{\sqrt{x+3}}}$$

$$\boxed{\frac{x^2+1}{3}}$$

SOLUTION:

$\frac{x+1}{\sqrt{x+3}}$ does not belong with the other three. The other three expressions are rational expressions. Since the denominator of $\frac{x+1}{\sqrt{x+3}}$ is not a polynomial, $\frac{x+1}{\sqrt{x+3}}$ is not a rational expression.

63. **REASONING** Determine whether the following statement is *sometimes, always,* or *never* true. Explain your reasoning.
A rational function that has a variable in the denominator is defined for all real values of x.

SOLUTION:

Sample answer: Sometimes; with a denominator like $x^2 + 2$, in which the denominator cannot equal 0, the rational expression can be defined for all values of x.

64. **OPEN ENDED** Write a rational expression that simplifies to $\frac{x-1}{x+4}$.

SOLUTION:

Sample answer: $\frac{x^2-1}{x^2+5x+4}$

65. **WRITING IN MATH** The rational expression $\frac{x^2+3x}{4x}$ is simplified to $\frac{x+3}{4}$. Explain why this new expression is not defined for all values of *x*.

SOLUTION:

Sample answer: When the original expression was simplified, a factor of *x* was taken out of the denominator. If *x* were to equal 0, then this expression would be undefined. So, the simplified expression is also undefined for *x*.

66. **SAT/ACT** The Mason family wants to drive an average of 250 miles per day on their vacation. On the first five days, they travel 220 miles, 300 miles, 210 miles, 275 miles, and 240 miles. How many miles must they travel on the sixth day to meet their goal?

A 235 miles

B 251 miles

C 255 miles

D 275 miles

E 315 miles

SOLUTION:

Let *x* miles must they travel on the sixth day to meet their goal.

$$\begin{aligned}\frac{220+300+210+275+240+x}{6} &= 250\\ \frac{1245+x}{6} &= 250\\ 1245+x &= 1500\\ x &= 1500-1245\\ x &= 255\end{aligned}$$

So, they must travel 255 miles on the sixth day to meet their goal. The correct choice is C.

67. Which of the following equations gives the relationship between N and T in the table?

N	1	2	3	4	5	6
T	1	4	7	10	13	16

F $T = 2 - N$

G $T = 4 - 3N$

H $T = 3N + 1$

J $T = 3N - 2$

SOLUTION:

The relationship between N and T in the table is $T = 3N - 2$. The correct choice is J.

68. A monthly cell phone plan costs $39.99 for up to 300 minutes and 20 cents per minute thereafter. Which of the following represents the total monthly bill (in dollars) to talk for x minutes if x is greater than 300?

A $39.99 + 0.20(300 - x)$

B $39.99 + 0.20(x - 300)$

C $39.99 + 0.20x$

D $39.99 + 20x$

SOLUTION:

If $x > 300$, then $x - 300$ represents the amount of minutes beyond the time included in the initial $39.99 charge. So, the total monthly bill (in dollars) to talk for x minutes is represented by the expression $39.99 + 0.20(x - 300)$. So, the correct choice is B.

69. **SHORT RESPONSE** The area of a circle 6 meters in diameter exceeds the combined areas of a circle 4 meters in diameter and a circle 2 meters in diameter by how many square meters?

SOLUTION:

Area of the circle of diameter 6 meters is 9π square meters.
Combined area of the circle of diameter 4 meters and the circle of 2 meters in diameter is $4\pi + \pi$ or 5π square meters.
$9\pi = 5\pi + 4\pi$
Therefore, the area exceeds by 4π square meters.

70. **ANTHROPOLOGY** An anthropologist studying the bones of a prehistoric person finds there is so little remaining Carbon-14 in the bones that instruments cannot measure it. This means that there is less than 0.5% of the amount of Carbon-14 the bones would have contained when the person was alive. The half-life of Carbon-14 is 5760 years. How long ago did the person die?

SOLUTION:

$$\begin{aligned} y &= ae^{-kt} \\ 0.5a &= ae^{-k(5760)} \\ 0.5 &= e^{-5760k} \\ \ln 0.5 &= \ln\left(e^{-5760k}\right) \\ \ln 0.5 &= -5760k \\ k &= \frac{\ln 0.5}{-5760} \\ &\approx 0.00012 \end{aligned}$$

Thus, the equation for the decay of Carbon-14 is $y = ae^{-0.00012t}$.

Substitute $0.005a$ for y in the equation $y = ae^{-0.00012t}$.

$$\begin{aligned} 0.005a &= ae^{-0.00012t} \\ 0.005 &= e^{-0.00012t} \\ \ln 0.005 &= \ln\left(e^{-0.00012t}\right) \\ \ln 0.005 &= -0.00012t \\ \frac{\ln 0.005}{-0.00012} &= t \\ t &\approx 44153 \end{aligned}$$

Therefore, the person died more than 44,000 years ago.

Solve each equation. Round to the nearest ten thousandth.

71. $3e^{x} + 1 = 5$

SOLUTION:

$$\begin{aligned} 3e^x + 1 &= 5 \\ 3e^x &= 4 \\ e^x &= \frac{4}{3} \\ \ln e^x &= \ln\frac{4}{3} \\ x &= \ln\frac{4}{3} \\ x &\approx 0.2877 \end{aligned}$$

72. $2e^{x} - 1 = 0$]

SOLUTION:

$$\begin{aligned} 2e^x - 1 &= 0 \\ 2e^x &= 1 \\ e^x &= \frac{1}{2} \\ e^x &= 0.5 \\ \ln e^x &= \ln 0.5 \\ x &= \ln 0.5 \\ x &\approx -0.6931 \end{aligned}$$

73. $-3e^{4x} + 11 = 2$

SOLUTION:

$$\begin{aligned} -3e^{4x} + 11 &= 2 \\ -3e^{4x} &= -9 \\ e^{4x} &= 3 \\ \ln e^{4x} &= \ln 3 \\ 4x &= \ln 3 \\ x &= \frac{\ln 3}{4} \\ x &\approx 0.2747 \end{aligned}$$

74. $8 + 3e^{3x} = 26$

SOLUTION:

$$\begin{aligned}
8 + 3e^{3x} &= 26 \\
3e^{3x} &= 18 \\
e^{3x} &= 6 \\
\ln e^{3x} &= \ln 6 \\
3x &= \ln 6 \\
x &= \frac{\ln 6}{3} \\
x &\approx 0.5973
\end{aligned}$$

75. **NOISE ORDINANCE** A proposed city ordinance will make it illegal in a residential area to create sound that exceeds 72 decibels during the day and 55 decibels during the night. How many times as intense is the noise level allowed during the day as at night?

SOLUTION:

The loudness L, in decibels, of a sound is $L = 10\ \log\frac{I}{m}$, where I is the intensity of the sound and m is the minimum intensity of sound detectable by the human ear.

Substitute 72 for L, m = 1 in $L = 10\ \log\frac{I}{m}$.

$$72 = 10\ \log\frac{I}{1}$$
$$72 = 10\ \log I$$
$$7.2 = \log I$$
$$I = 10^{7.2}$$

Now, substitute 55 for L, m = 1 in $L = 10\ \log\frac{I}{m}$.

$$55 = 10\ \log\frac{I}{1}$$
$$55 = 10\ \log I$$
$$5.5 = \log I$$
$$I = 10^{5.5}$$
$$\frac{10^{7.2}}{10^{5.5}} = 10^{7.2-5.5}$$
$$= 10^{1.7}$$

Therefore, the noise level allowed during the day is $10^{1.7}$ or about 50 times of the noise level allowed during the night.

Simplify.

76. $\sqrt{50x^4}$

SOLUTION:

$$\sqrt{50x^4} = \sqrt{2\cdot(25x^4)}$$
$$= \sqrt{2\cdot(5x^2)^2}$$
$$= 5x^2\sqrt{2}$$

8-1 Multiplying and Dividing Rational Expressions

77. $\sqrt[3]{16y^3}$

SOLUTION:

$$\begin{aligned}\sqrt[3]{16y^3} &= \sqrt[3]{2(8y^3)}\\ &= \sqrt[3]{2(2y)^3}\\ &= 2y\sqrt[3]{2}\end{aligned}$$

78. $\sqrt{18x^2y^3}$

SOLUTION:

$$\begin{aligned}\sqrt{18x^2y^3} &= \sqrt{2y(9x^2y^2)}\\ &= \sqrt{2y(3xy)^2}\\ &= 3|x|y\sqrt{2y}\end{aligned}$$

79. $\sqrt{40a^3b^4}$

SOLUTION:

$$\begin{aligned}\sqrt{40a^3b^4} &= \sqrt{4\cdot 10a^3b^4}\\ &= 2ab^2\sqrt{10a}\end{aligned}$$

80. **AUTOMOBILES** The length of the cargo space in a sport-utility vehicle is 4 inches greater than the height of the space. The width is 16 inches less than twice the height. The cargo space has a total volume of 55,296 cubic inches.

a. Write a polynomial function that represents the volume of the cargo space.

b. Will a package 34 inches long, 44 inches wide, and 34 inches tall fit in the cargo space? Explain.

SOLUTION:

a. Let *h* be the height of the cargo space. Therefore, the length of the space is $(h + 4)$ and the width of the space is $(2h - 16)$.

$$\begin{aligned} V &= (h+4)(2h-16)h \\ &= (2h^2-16h+8h-64)h \\ &= (2h^2-8h-64)h \\ &= 2h^3-8h^2-64h \end{aligned}$$

A polynomial function that represents the volume of the cargo space is $V = 2h^3 - 8h^2 - 64h$.

b. Substitute 55296 for *V* in the equation $V = 2h^3 - 8h^2 - 64h$.

$$55296 = 2h^3 - 8h^2 - 64h$$

$$2h^3 - 8h^2 - 64h - 55296 = 0$$

$$\begin{array}{r|rrrr} 32 & 2 & -8 & -64 & -55296 \\ & 0 & 64 & 1792 & 55296 \\ \hline & 2 & 56 & 1728 & \lfloor 0 \end{array}$$

So, $h = 32$ is a root of the equation $2h^3 - 8h^2 - 64h - 55296 = 0$.
The depressed polynomial is $(2h^2 + 56h + 1728)$.
Therefore, $2h^3 - 8h^2 - 64h - 55296 = (h-32)(2h^2 + 56h + 1728)$.

Use the Quadratic formula to find the roots of $2h^2 + 56h + 1728 = 0$.

$$\begin{aligned} h &= \frac{-56 \pm \sqrt{56^2 - 4(2)(1728)}}{2(2)} \\ &= \frac{-56 \pm \sqrt{3136 - 13824}}{4} \end{aligned}$$

Therefore, the other two roots of the equation $2h^3 - 8h^2 - 64h - 55296 = 0$ are imaginary. The length of the space is 36 inches. The width of the space is 48 inches. So, the package is too tall to fit.

Simplify.

81. $(2a + 3b) + (8a - 5b)$

SOLUTION:

$$2a + 3b + 8a - 5b = 10a - 2b$$

82. $(x^2 - 4x + 3) - (4x^2 + 3x - 5)$

SOLUTION:

$x^2 - 4x + 3 - 4x^2 - 3x + 5 = -3x^2 - 7x + 8$

83. $(5y + 3y^2) + (-8y - 6y^2)$

SOLUTION:

$5y + 3y^2 - 8y - 6y^2 = -3y - 3y^2$

84. $2x(3y + 9)$

SOLUTION:

$2x(3y + 9) = 6xy + 18x$

85. $(x + 6)(x + 3)$

SOLUTION:

$$\begin{aligned}&(x+6)(x+3)\\&=x^2+6x+3x+18\\&=x^2+9x+18\end{aligned}$$

86. $(x + 1)(x^2 - 2x + 3)$

SOLUTION:

$$\begin{aligned}&(x+1)(x^2-2x+3)\\&=x^3-2x^2+3x+x^2-2x+3\\&=x^3-x^2+x+3\end{aligned}$$